Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math
Spring 2015

Name:______________________________

Instructors: Garbett & Migliore

Exam III Solutions

April 16, 2015

This exam is in two parts on 12 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You may use a calculator, but no books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

You must record on this page your answers to the multiple choice problems.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

Place an × through your answer to each problem.

1.	(a)	(b)	(c)	(d)	(e)
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Multiple Choice

1. (5 pts.) There are five lighthouses in the state of Alabama. Their heights (in feet) are

45, 63, 41, 49, 132

The mean height is 66 feet (you don't have to verify this). What is the (population) variance for the heights of lighthouses in Alabama?

(a) 1430 (b) 37.82 (c) 1144

(d) 33.82 (e) 66

Solution:

There are 5 heights, so

$$\sigma = \frac{(45-66)^2 + (63-66)^2 + (41-66)^2 + (49-66)^2 + (132-66)^2}{5} = 1144$$

2. (5 pts.) The scores for an exam worth 100 points have mean 80 and standard deviation 10. If the scores are normally distributed, what percentage of students got an A (had a score greater than 90) on the exam?

(a) 15.87% (b) 0% (c) 50%

(d) 34.13% (e) 84.13%

Solution:

Let X =student's score. First, we compute the z-score for 90

$$\frac{90-80}{10} = \frac{10}{10} = 1$$

Therefore,

$$P(X > 90) = P(Z > 1) = 1 - P(Z \leq 1) = 1 - 0.8413 = 0.1587$$

where we used the standard normal table to find $P(Z \leq 1)$. Thus, 15.87% of students got an A.

3. (5 pts.) Adam rolls a 4-sided die 8 times. What is the probability that he rolls at least 6 fours?

(a) $C(8,6)(0.25)^6(0.75)^2$

(b) $C(8,6)(0.25)^6(0.75)^2 \cdot C(8,7)(0.25)^7(0.75)^1 \cdot C(8,8)(0.25)^8(0.25)^0$

(c) $C(8,6)(0.75)^6(0.25)^2 + C(8,7)(0.75)^7(0.25)^1 + C(8,8)(0.75)^8(0.25)^0$

(d) $C(8,6)(0.75)^6(0.25)^2$

(e) $C(8,6)(0.25)^6(0.75)^2 + C(8,7)(0.25)^7(0.75)^1 + C(8,8)(0.25)^8(0.25)^0$

Solution:
This is a Bernoulli experiment with 8 trials. A success is getting a 4, and a failure is not getting a 4. We have $n = 8$, $p = P(\text{success}) = 0.25$, and $q = P(\text{failure}) = 0.75$. Let X be the number of fours Adam rolls. Since X is a binomial random variable, the probability of getting at least 6 fours is

$$\begin{aligned} P(X \geq 6) &= P(X = 6) + P(X = 7) + P(X = 8) \\ &= C(8,6)(0.25)^6(0.75)^2 + C(8,7)(0.25)^7(0.75)^1 + C(8,8)(0.25)^8(0.25)^0 \end{aligned}$$

4. (5 pts.) Let Z be a standard normal random variable. Find the number x so that

$$P(-1 \leq Z \leq x) = 0.4967.$$

(a) 0.1587 (b) 0.4 (c) 0.4967

(d) 0.6554 (e) 1

Solution:
Notice that

$$P(-1 \leq Z \leq x) = P(Z \leq x) - P(Z \leq -1) = P(Z \leq x) - 0.1587$$

where we got $P(Z \leq -1) = 0.1587$ from the standard normal table. Therefore,

$$P(Z \leq x) - 0.1587 = P(-1 \leq Z \leq x) = 0.4967.$$

Adding 0.1587 to both sides gives

$$P(Z \leq x) = 0.4967 + 0.1587 = 0.6554.$$

Now we find 0.6554 in the standard normal table to find $x = 0.4$.

5. (5 pts.) The following is the probability distribution for a random variable, X,

k	P(X=k)
0	0.3
1	0.5
2	0.1
3	0.1

What is $\sigma(X)$ (rounded to 3 decimal places)?

(a) 0.800 (b) 0.447 (c) 0.894

(d) 1.225 (e) 1.500

Solution:
First, we must fine $E(X)$, which is

$$E(X) = 0 \cdot 0.3 + 1 \cdot 0.5 + 2 \cdot 0.1 + 3 \cdot 0.1 = 1.$$

Now we have

$$\sigma^2(X) = (0-1)^2 \cdot 0.3 + (1-1)^2 \cdot 0.5 + (2-1)^2 \cdot 0.1 + (3-1)^2 \cdot 0.1 = 0.3 + 0 + 0.1 + 0.4 = 0.8.$$

Thus, $\sigma(X) = \sqrt{0.8} = 0.894$.

6. (5 pts.)
Consider the system of inequalities

$$\begin{array}{rcrcl} x & + & y & \leq & 5 \\ x & - & 2y & \leq & 2 \\ & & x & \geq & 1 \end{array}$$

Which of the following points is in the feasible set defined by these inequalities? (Notice that we are not asking you to graph the feasible set.)

(a) $(3,3)$ (b) $(0,4)$ (c) $(5,1)$

(d) $(2,-1)$ (e) $(1,2)$

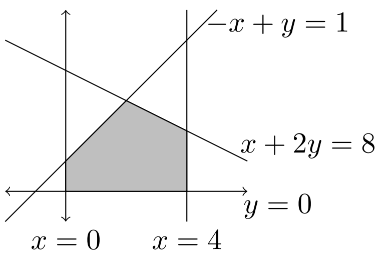
Solution:
The answer is (e). Plugging in $x = 1, y = 2$ in all three inequalities gives correct statements, while any other choice from among the possible answers gives at least one inequality that's not satisfied.

7. (5 pts.) Consider the system of inequalities

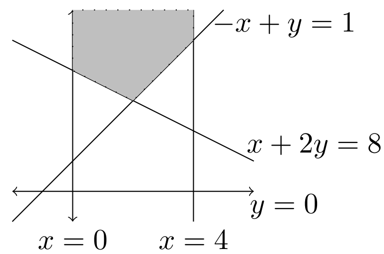
$$\begin{array}{rcrcl} -x & + & y & \geq & 1 \\ x & + & 2y & \leq & 8 \\ & & x & \leq & 4 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

Which of the following shaded regions is the feasible set for this system of linear inequalities?

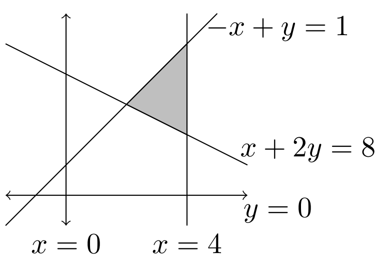
(a)



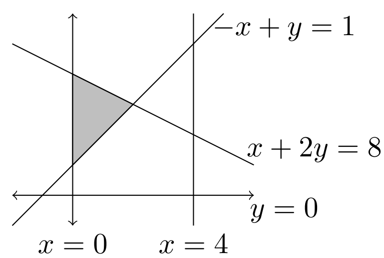
(b)



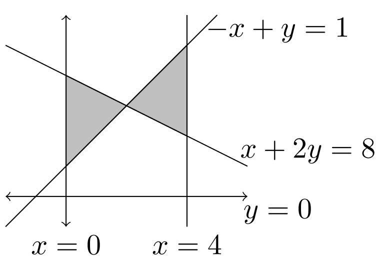
(c)



(d)



(e)



Solution:

The answer is (d). Each of the other choices violates one or more of the inequalities.

8. (5 pts.) Bob and Bill's Glassworks is a store that makes and sells glass lamps and crystal balls. Bob makes the items and Bill puts in the finishing touches. Each glass lamp takes 2 hours for Bob to make and 1 hour for Bill to put finishing touches on. Each crystal ball takes 4 hours for Bob to make and 2 hours for Bill to put finishing touches on. Bob works 50 hours every week and Bill works 40 hours every week. Let x be the number of glass lamps made and y the number of crystal balls made. Which of the following is one of the inequalities that come from this information? (**Pay attention to $\leq$ versus $\geq$.** There is no objective function in this problem.)

(a) $x + 2y \leq 40$ (b) $x + 2y \geq 40$ (c) $2x + y \leq 40$

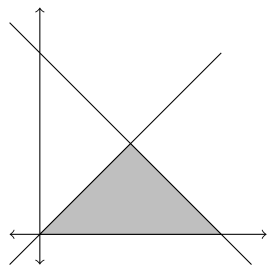
(d) $2x + y \geq 40$ (e) $2x + 4y \geq 50$

Solution:

The inequality for Bob is
$$2x + 4y \leq 50.$$
The inequality for Bill is
$$x + 2y \leq 40.$$
We also have $x \geq 0$ and $y \geq 0$. So the answer is (a).

9. (5 pts.)
Consider the system of inequalities whose feasible set is sketched to the right.

$$\begin{array}{rcrcl} -x & + & y & \leq & 0 \\ x & + & y & \leq & 6 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$



What is the maximum value of the objective function $2x + 3y$ subject to the above constraints?

(a) 0 (b) 12 (c) 18 (d) 15

(e) There is no maximum value with these constraints and objective function.

Solution:
The vertices are easily computed to be $(0,0)$, $(3,3)$ and $(6,0)$. Plugging these values into the objective function gives, respectively, values of 0, 15 and 12. So the maximum value is 15.

10. (5 pts.) Find the following matrix product:

$$\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & -1 \end{bmatrix}$ (e) This product is impossible

Solution:
The answer is (c), by matrix multiplication. Notice that we have a

$$(2 \times 2) \cdot (2 \times 3) \cdot (3 \times 1) = (2 \times 1)$$

matrix for the answer.

Partial Credit

You must show all of your work on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (10 pts.) Sunny brand orange juice is produced in 40 oz bottles. However, the machines that dispense the orange juice into the bottles are not able to dispense exactly 40 oz of orange juice each time. The amount of juice dispensed is normally distributed with mean 40.5 (ounces) and standard deviation 5 (ounces).

(a) What percentage of bottles of Sunny brand orange juice produced contain less than the advertised amount of juice (less than 40 ounces)?

Solution:
Let X be the amount of juice dispensed. First, we compute the z-score:

$$\frac{40-40.5}{5} = \frac{-0.5}{5} = -0.1$$

Therefore, we have

$$P(X < 40) = P(X < -0.1) = 0.4602.$$

Thus, 46.02% of the bottles produced contain less than 40 ounces of juice.

(b) What percentage of bottles of Sunny brand orange juice produced contain between 39 and 41 ounces of juice?

Solution:
We again begin by computing z-scores.

$$\frac{39-40.5}{5} = \frac{-1.5}{5} = -0.3$$
$$\frac{41-40.5}{5} = \frac{0.5}{5} = 0.1$$

Therefore,

$$\begin{aligned} P(38 \le X \le 41) = P(-0.3 \le Z \le 0.1) &= P(Z \le 0.1) - P(Z \le -0.3) \\ &= 0.5398 - 0.3821 = 0.1577, \end{aligned}$$

so 15.77% of bottles contain between 39 and 41 ounces of juice.

12. (10 pts.) My dog loves to play fetch. I know from past experience that he catches the ball in the air 80 percent of the time (so he misses 20 percent of the time). Suppose I throw the ball 10 times. Let X be the number of times my dog catches the ball. You do not need to simplify your answers in parts (a) or (b).

(a) What is the probability that my dog catches the ball exactly 7 times?

Solution:
The random variable X is a binomial random variable with 10 trials. We have $p = 0.8$ and $q = 0.2$, so the probability that my dog catches the ball exactly 7 times is

$$P(X = 7) = C(10, 7)(0.8)^7(0.2)^3.$$

(b) What is the probability that my dog catches the ball between 4 and 7 times, inclusive? (That is, compute $P(4 \le X \le 7)$.)

Solution:

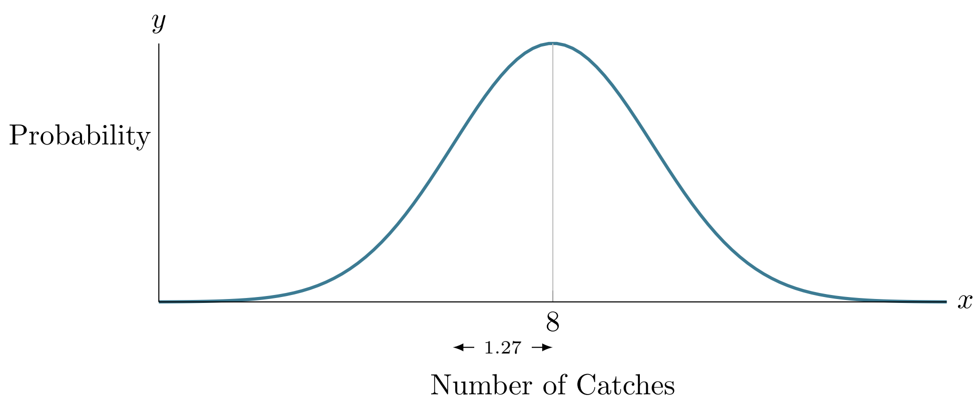
$$P(4 \le X \le 7) = C(10,4)(0.8)^4(0.2)^6 + C(10,5)(0.8)^5(0.2)^5 + C(10,6)(0.8)^6(0.2)^4 + C(10,7)(0.8)^7(0.2)^3$$

(c) Sketch the normal curve that best approximates the probability distribution of the random variable X. Be sure to include labeled axes and a scale. Also, give the mean and standard deviation of the normal curve you're drawing. (Hint: The mean and standard deviation should be the mean and standard deviation for X. You should simplify your answer here.)

Solution:
First we compute the mean and standard deviation of X. We have

$$\mu = np = (10)(0.8) = 8$$
$$\sigma = \sqrt{npq} = \sqrt{(10)(0.8)(0.2)} \approx 1.27$$

The normal curve that best approximates the probability distribution of X has the same mean and standard deviation as X. Therefore, we have



13. (10 pts.) Tom pays 5 dollars to play the following game. First, he rolls a 4-sided die. The number he rolls gives the number of dollars he wins. If Tom rolls a 2, 3, or 4, the game is over. If he rolls a 1, he gets to flip a coin. If the coin flip results in heads, he gets 10 dollars more, and the game ends. If the coin flip results in tails, he wins no additional money, and the game ends. Let X be Tom's net earnings.

(a) Give a probability distribution for X. (A tree diagram might be helpful for determining the probabilities.)

Solution:
Since we're interested in Tom's net earnings, we must take the amount he pays to play the game into account. This means the possible values for X are -3 (when Tom rolls a 2), -2 (when Tom rolls a 3), -1 (when Tom rolls a 4), -4 (when Tom rolls a 1 and then gets tails on the coin), 6 (when Tom rolls a 1 and then gets heads on the coin). We get the following probability distribution.

k	P(X=k)
-4	1/8
-3	1/4
-2	1/4
-1	1/4
6	1/8

(b) What is $E(X)$?

Solution:

$$E(X) = (-4)(1/8) + (-3)(1/4) + (-2)(1/4) + (-1)(1/4) + (6)(1/8) = -1.25$$

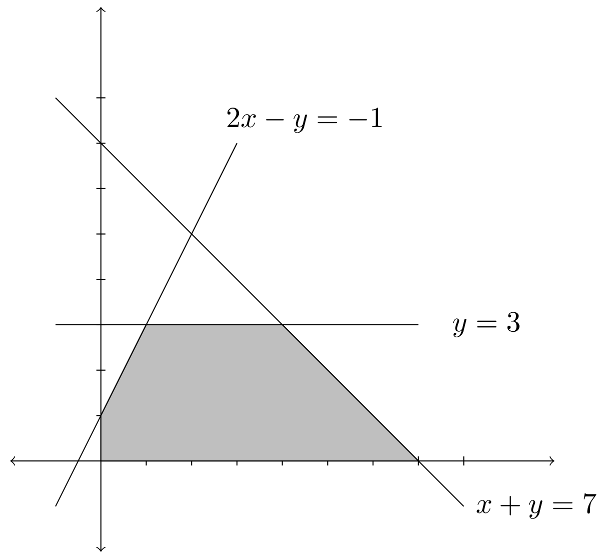
(c) If Tom plays the game 100 times, should he expect to win money or lose money? How much should he expect to win or lose? Explain your answer.

Solution:
We showed in part (b) that on average Tom loses \$1.25 per game. Therefore, in 100 games, he would expect to lose about $-\$1.25 \cdot 100 = \$125$.

14. (10 pts.)

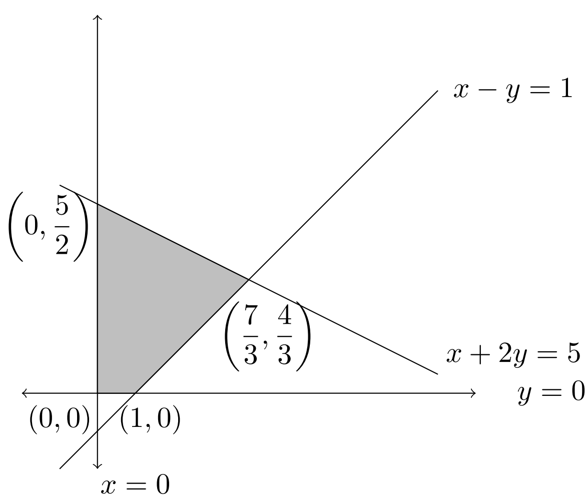
(a) Graph the feasible set of the region given by the given inequalities using the axes provided. Be sure to shade in the entire feasible set (and nothing else). In this part of the problem we are not asking you to label the corners of the feasible set, but please label your lines.

$$\begin{aligned} 2x - y &\geq -1 \\ x + y &\leq 7 \\ x &\geq 0 \\ y &\leq 3 \\ y &\geq 0 \end{aligned}$$



Solution:
The answer is given in the axes above.

(b) Find the coordinates of the corners of the following feasible set. You can put your answers in the figure itself.



Solution:
The answer is given in the figure.

15. (10 pts.)

Bob is taking a French class. The students have to read some combination of books and magazines. Each magazine is 100 pages long and takes 3 hours to read. Each book is 200 pages long and takes 5 hours to read. The requirement is that each student read at least 12,000 pages over the course of the semester. Bob wants to minimize the amount of time he spends reading for this class. For this problem we'll let x be the number of magazines he reads and y be the number of books he reads.

(a) Find the constraints for this problem. (That is, find the inequalities.)

Solution:

$$\begin{aligned} 100x + 200y &\geq 12,000 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

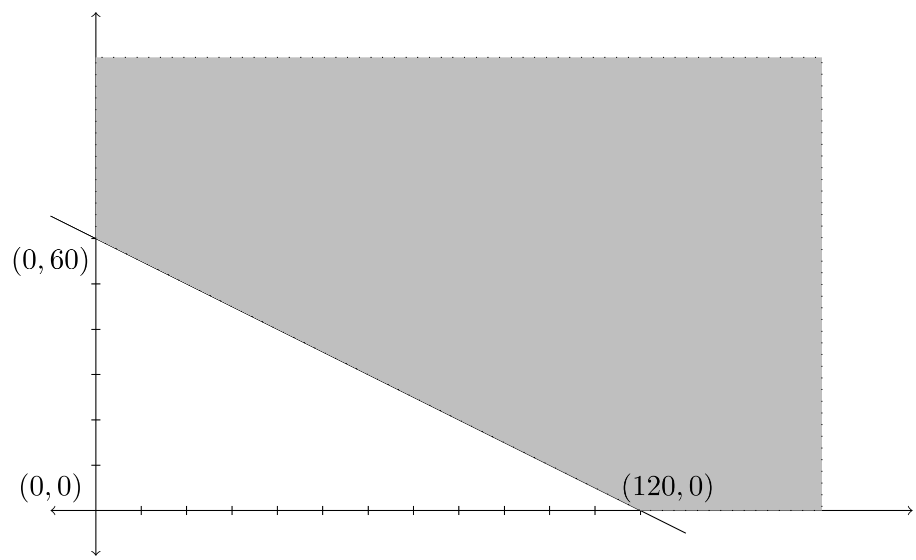
(b) Find the objective function.

Solution:

$3x + 5y$.

(c) Graph the inequalities and shade the feasible set. This includes finding the coordinates of the corners.

Solution:



(d) In order to minimize the amount of time that he spends reading for the class, how many magazines and how many books should Bob read?

Solution:

Plugging $(0, 60)$ into the objective function gives a value of 300. Plugging in $(120, 0)$ gives a value of 360. Since he wants to minimize the number of hours, he should read 0 magazines and 60 books.

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